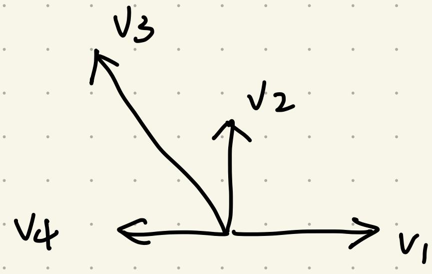
① What is a matroid

key: abstracts linear independece.

(start defin by an example) 3.4.2.1

$$\begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ A = & \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & 4 & 0 \end{bmatrix} \end{array}$$



$\downarrow$

matroid M_A

zero vec overline.

all the way underline.

equivalent data of $\{1,2\}$ $\parallel$

① bases $\mathcal{B}(M_A) = \{12, 13, 23, 24, 34\}$

set of basis of span(v_1, v_2, v_3, v_4)

(put index of basis into a set)

→ rank.

② independent set $\mathcal{I}(M_A) = \{12, 13, 23, 24, 34, 1, 2, 3, 4, \phi\}$

has sets of linearly ind vectors' indexes

Subsets of bases

③ circuits $\mathcal{C}(M_A) = \{14, 123, 234\}$

sets of minimally dependent sets

(if delete any vector, the remaing ones are linearly independent

(equivalent $\Leftrightarrow$ bijections between any two)

10mins.

(Notice we haven't formally defined matroid)

Def A matroid $M = (E, \mathcal{B})$ is a pair where E is a finite set (ground set) $E = [n] = \{1, \dots, n\}$, and $\mathcal{B}$ is a family of subsets of E (bases) s.t.

(1) $\mathcal{B} \neq \phi$

(2) basis exchange property

$B_1, B_2 \in \mathcal{B}$ and $x \in B_1 - B_2$

$\Rightarrow \exists \ y \in B_2 - B_1$ s.t. $(B_1 \setminus \{x\}) \cup \{y\} \in \mathcal{B}$.

(since bijection, imagine $\exists$ other defin using ind sets / circuits) take-away : B & E enough.

(one more thing then ex)

Def ? A matroid comes from a matrix A is realizable / representable

(ellaboration on representable) ?

check ex is matroid

17 min

Def P, Q matroids

P is a quotient of Q if any circuit of Q is a union of circuits of P

Def A positroid $M = M_A$ is a realizable matroid from a $k \times n$ matrix A of rank k all real entries, and all <u>maximal minors</u> are non-negative.

determinant of $k \times k$ submatrices

rank of M_A = rank of A = card of a basis (dim of span col vectors)

ex. $\Delta_{12} = 1$ $\quad \Delta_{13} = 4 \quad \Delta_{14} = 0$

col index

$\Delta_{23} = 3 \quad \Delta_{24} = 1 \quad \Delta_{34} = 4$

!! we don't like the definition

prob: easier way to count(?) positroids

open question: when are P, Q
quotients?

partially answered by:

Benedetti - Chavez - Tamayo 19'

if Q uniform (class of positroid)

$rk(Q) = rk(P) + 1$

VS:
when the larger one is uniform matroid,
we give complete characterization for this.

Benedetti - Kraner 22'

if P, Q LPM (lattic path)

complete characterization

posi
↓
LPM
↓
uniform.

VS:
different proof using necklace.

our work: when rank of positroids differ
by 1, gave necessary
condition for this

30 mins.

② decorated permutation

Def A <u>decorated permutation</u> on $[n]$ is a pair (π, col), where π is a permutation, and $col: [n] \to \{-1, 0, 1\}$, s.t. $f^{-1}(\{0\})$ has all unfixed point.

Notation. (only work on decorated perm)

$col(x) = 1$ overline

-1 underline.

ex.

$$\pi = 41\bar{3}562\underline{7}$$

Thm Postnikov 06

bijection between positroids & decorated perms

Take-away

index positroids by decorated permutations

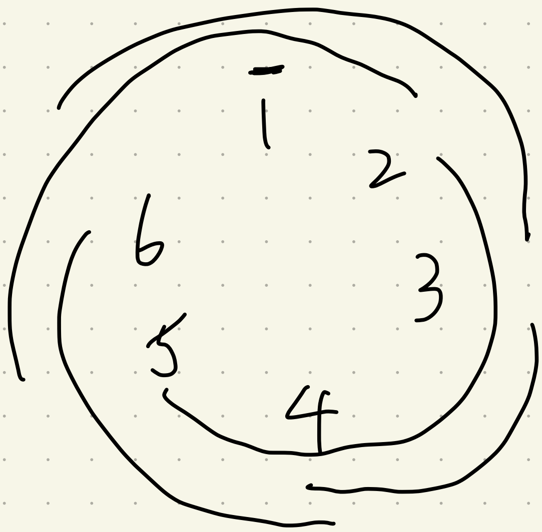
(M_π, P_σ, Q_π).

Def A uniform matroid on $[n]$ of rank k is $U_{k,n} = ([n], \mathcal{B} = \binom{[n]}{k})$ all k-element subsets of $[n]$

Subclass of positroid.

Def CW-arrow (by example).

$\pi = \bar{1}54623$



overline : singleton

underline : $[n]$

1, 2345, 34, 456

5612, 6123

7 min.

Theorem 3 (Chen - Fei - Gao - S. - Zhang).

fix integers $0 \le r \le k < n$. Let P be a positroid of rank $k-r$ on $[n]$. Then P is a quotient of $U_{k,n}$ iff the union of any $r+1$ CW-arrows of P has cardinality at least $k+1$

ex. π rank $= 2$

$U_{4,6}$

$n=6$, $k=4$, $rk(\pi) = k - r$, $r = 2$.

Not quotient

· Q $\overline{6\,2\,3\,4\,5}\,1$ rank 1

$123456, 2, 3, 4, 5, 61$