Crystal Chute Moves on Pipe Dreams

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Joint work with Elizabeth Milićević & Sarah Gold

TLDR: only look at a kind of chute moves

rep/geo? ⇒ they form a Demazure crystal on PDs

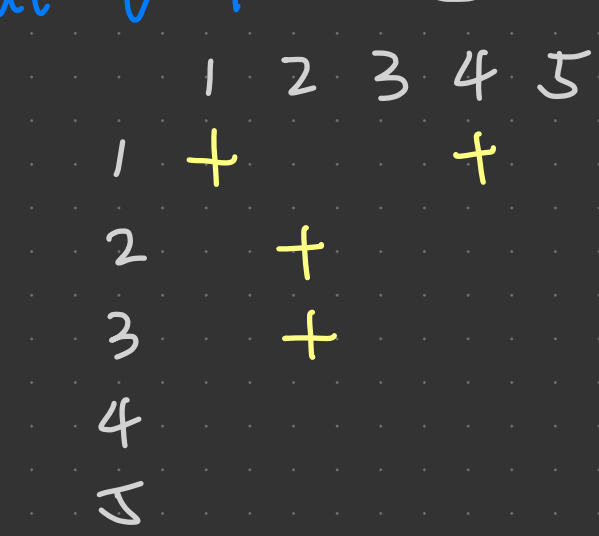
Schubert sem ⇒ $\mathfrak{S}$ decomposes as key polynomials

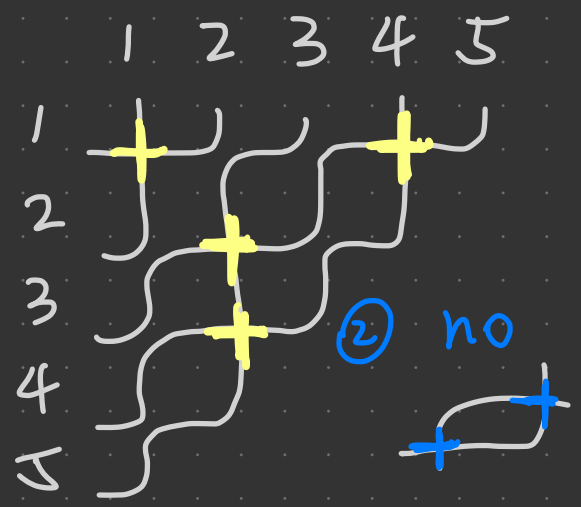
Reduced Pipe Dreams

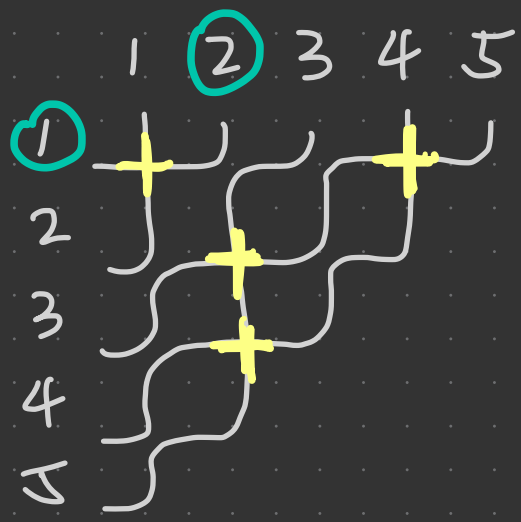
Pick a permutation $w \in S_n$ of length ℓ

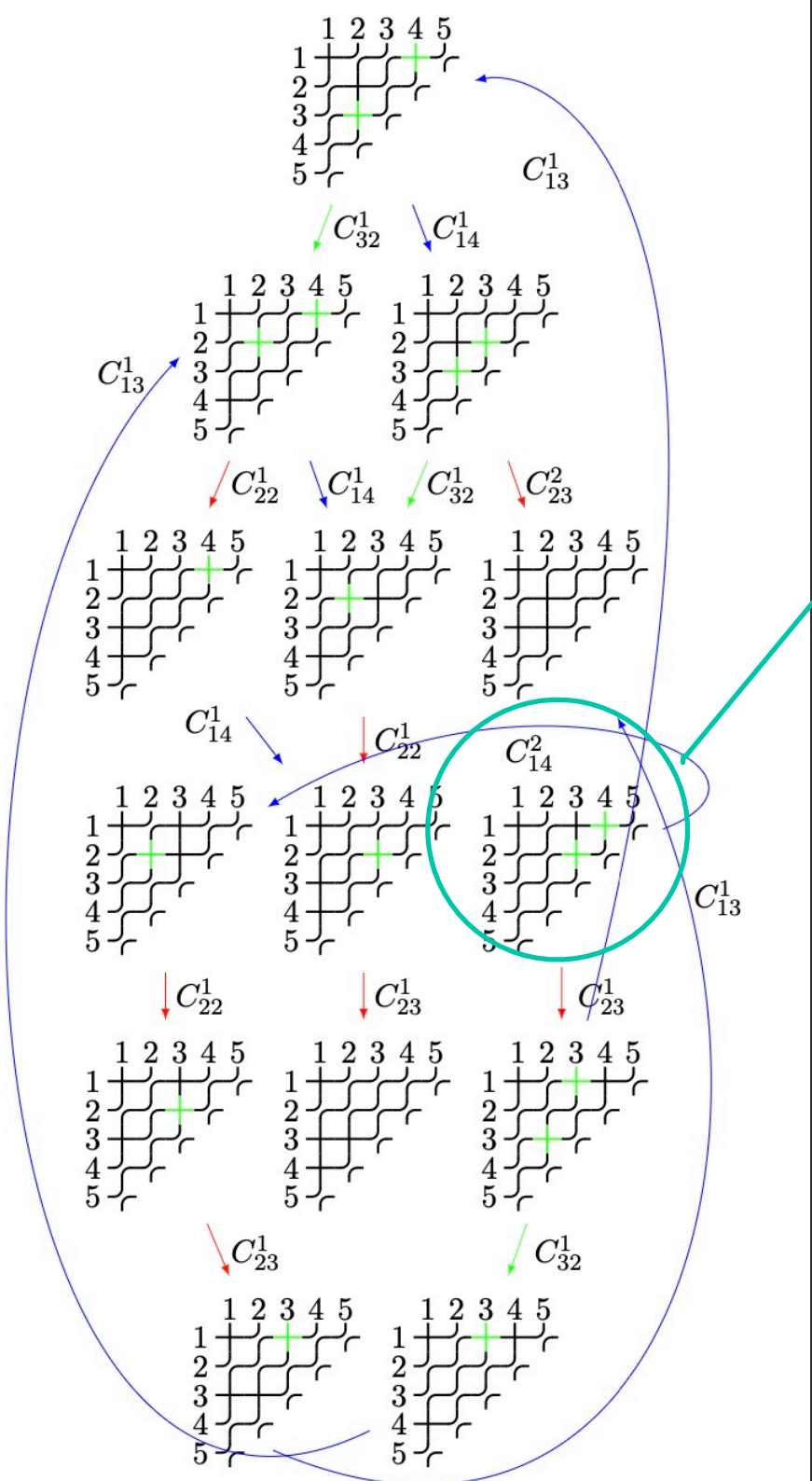
say $w = [21543] \in S_5$ (length 4)

① put ℓ +



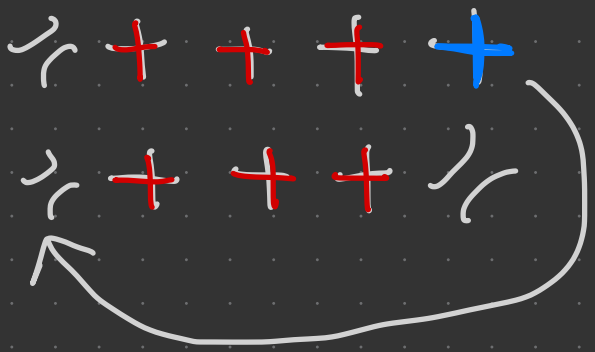






← all PDs of [21543]

generates all PDs using chute moves



Pairing Process

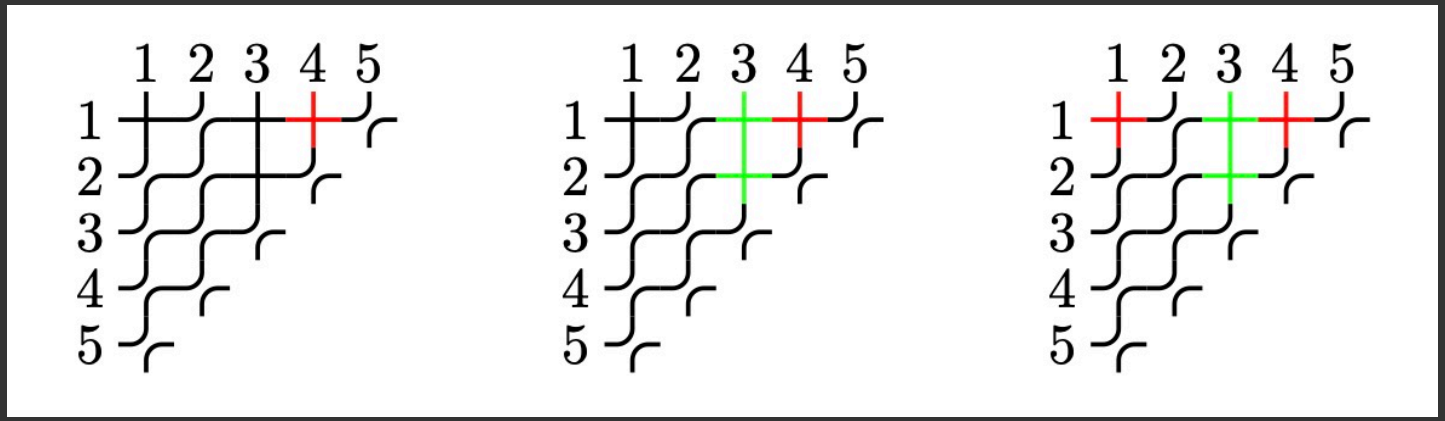
given a PD, a pairing process on row i works as:

① start at the rightmost cross c at row i

② find a cross c' at row $i+1$ s.t. it's weakly right of c

ⓐ if $\exists$, say c and c' are paired

ⓑ otherwise, c is unpaired

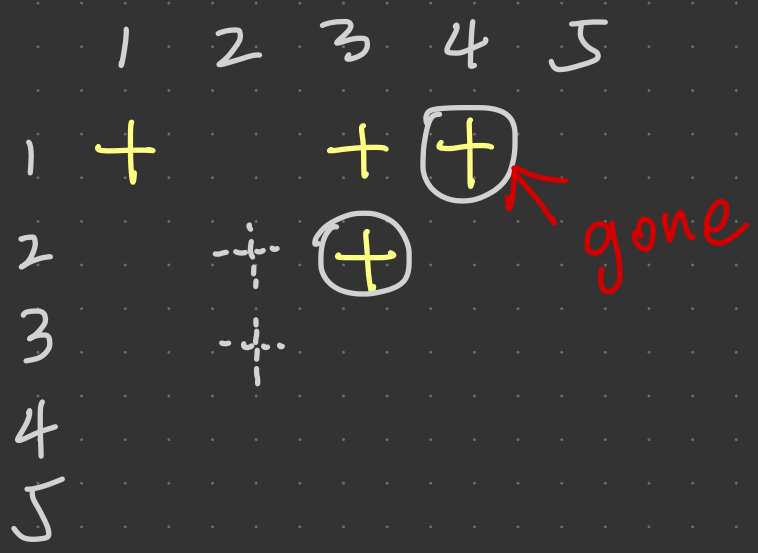


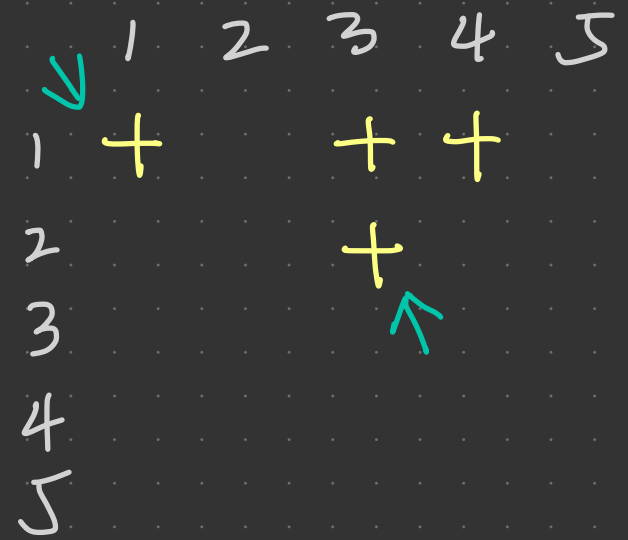
Crystal Operators

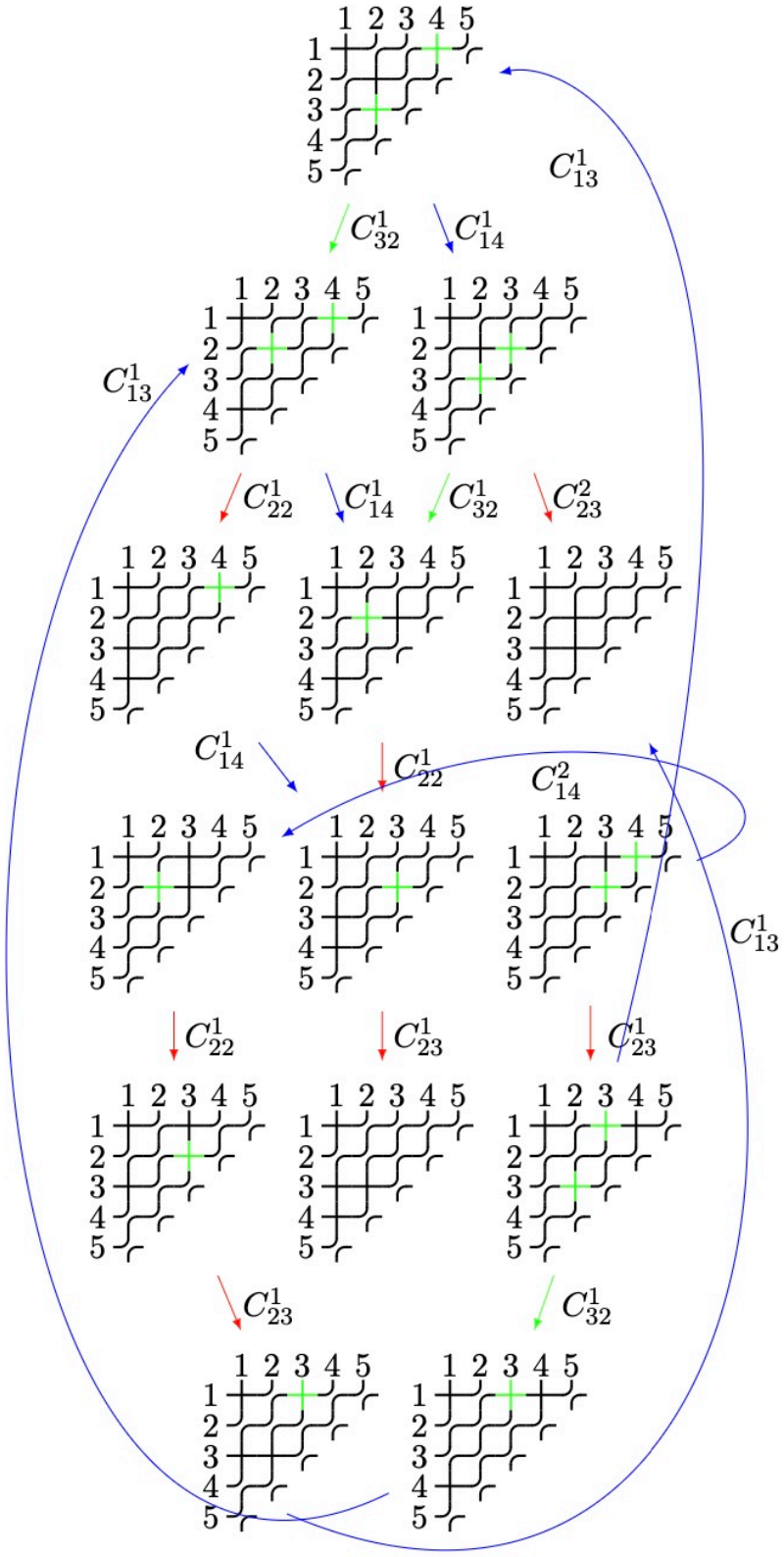
Def a lowering operator f_i on a PD

① a chute move on a cross at row i

② and the cross is the leftmost unpaired cross after running a pairing process at row i

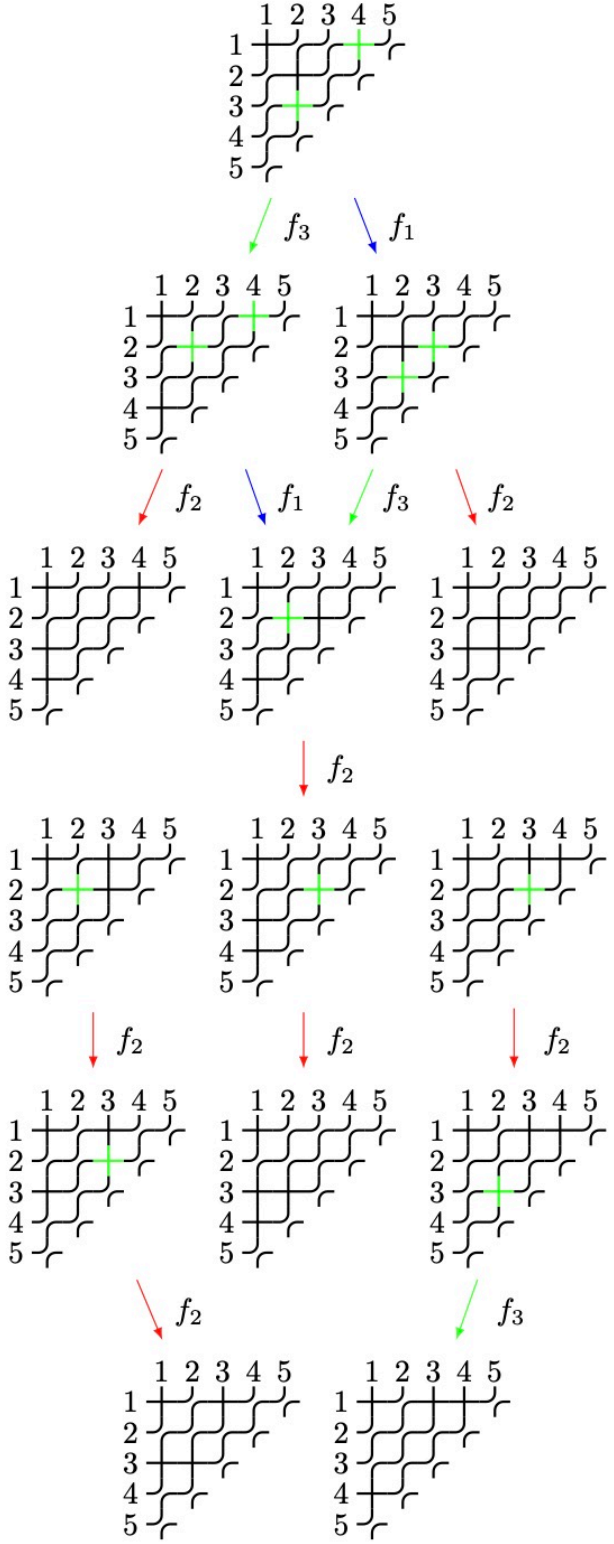






$e_i(PD) = 0$

if all crosses at row $i+1$ are paired after pairing process at row i



Thm (Gold-Miličević-S)

Let $w \in S_n$. The operators e_i, f_i for $1 \le i < n$ define a type A_{n-1} Demazure crystal structure on $\{$ reduced PDs of $w \}$

$$RPD(w) = \bigcup_{D \in RPD(w),\ e_i(D) = 0,\ \forall 1 \le i < n} B_{\pi_D}(wt(D))$$

π_D: a perm uniquely determined by D.

$wt(D)$: # of i in partition for highest weight

highest weight

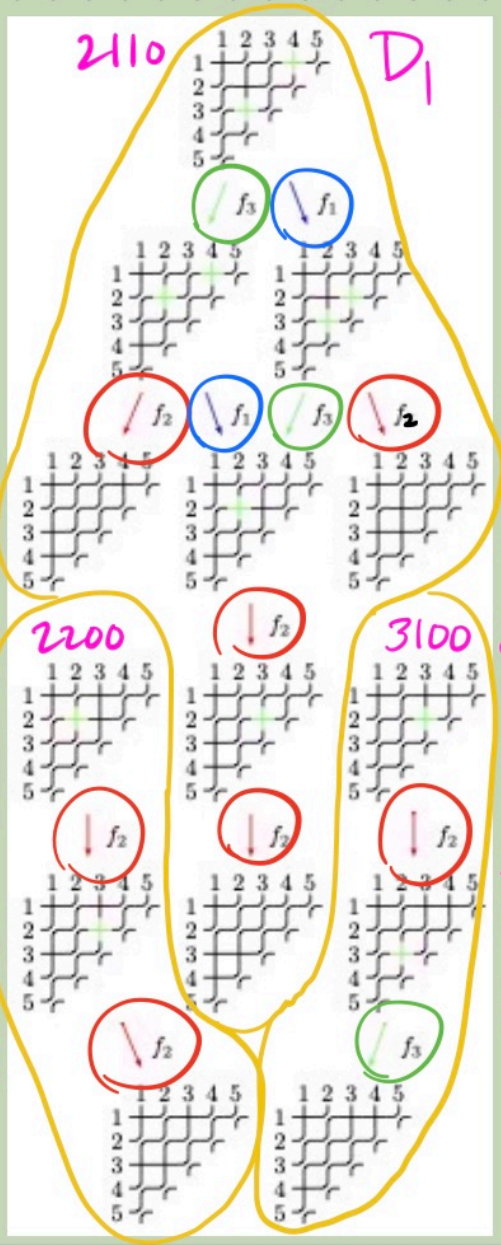
Related:

Assaf - Schilling (RFC)

Lenart (coplactic operators biwords)

Crystals + Pipe Dreams

Example: $w = [21543]$



$\pi_{D_1} = s_2 s_1 s_3$

$1021 = s_{213}(2110)$

$\pi_{D_2} = s_2$

$2020 = s_2(2200)$

$\pi_{D_3} = s_3 s_2$

$3001 = s_{32}(3100)$

Corresponding Key Polynomials are indexed by the composition

$$\alpha_D := \pi_D(\mathrm{wt}(D))$$

$$\mathfrak{S}_{[21543]} = K_{(1021)} + K_{(2020)} + K_{(3001)}$$

How to get π_D ?

① what we just did

② RFC → Edelman-Greene insertion → lift (Assaf & Schilling)

③ algorithm at sec 6 (could skip insertion)

④ etc ...

Follow-up ? chute move connecting keys ?

Thank You !