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Logic Paper 3 on Wittgenstein

Look at What's Really There!

We use names and concepts to refer to objects, but we could never reach the object completely. We use logical connectives to form propositions, but since our final goal is the truth values, we could just observe the truth table. We use laws of inference to draw conclusions, but all was hidden in the premises originally. Wittgenstein provides perspectives on the observation of objects, truth-possibilities and inference that aim to directly reach our intention of having the correct conclusion.

Wittgenstein's system shares a similar pattern with Kant's system where facts correspond to judgments and names intuitions. We could recall that in Kant's system, intuitions give objects and we could only think about objects through concepts. At the same time, in Wittgenstein's system, after being labelled by names, objects could be thought or displayed in different ways. For instance, if I say Felix is a cat, then "is a cat" is a predicate; "cat" is a concept we invented in biology and what we use to think about Felix; Felix is named as Felix and is displayed as a cat in this sentence. In Wittgenstein's system, this is a fact. Facts are states of affairs, and states of affairs are combinations of objects(2.01). To be a fact, it needs to have objects that are displayed in a certain way. This definition of facts resembles the definition of pictures, whose elements are related to one another in a determinate way(2.14). Thus a picture is a fact(2.141). One might notice that this sentence is a judgement, which is the smallest unit of cognition in Kant's system. Since Wittgenstein says the world is the totality of facts(2.04), a fact seems to also be the smallest unit of cognition in his system. Furthermore, a fact is more flexible than a judgment when it comes to possible ways to express it. What Wittgenstein adds is that there are many other ways we could display the fact. For example, if we draw two cat ears at the top of the word "Felix", whoever reads it will probably be able to guess that it means Felix is a cat. Using a predicate, using a concept, or using relation letters are just some of the possible ways of displaying the object. Overall, although significant parts of Wittgenstein's and Kant's systems are parallel to each other, Wittgenstein's definition of facts seems to be a bit more ambitious than Kant's.

Before entering an intriguing discussion about propositions, some fundamental knowledge is necessary. An analogy I use to understand it is to think about a proposition as a theme in music. Elements in a proposition with a spatial arrangement to the proposition are like musical notes to the theme in music. Thus every proposition must already have a sense(4.064), as the theme in music is based on the well-structured arrangement of the notes. The propositional sign, which is the projective relation of the proposition to the world, is like the performance of the music(3.12). Then it makes more sense to consider a propositional sign as a fact, since it could be perceived as a more "down to earth" version of the proposition, as the performance is more real and actual than musical notes(3.14). Now if we think about "the logical picture" of a performance, we could refer back to the theme and the musical notes. In parallel, the logical picture of a fact is a thought, and a thought is a proposition with sense(3). Overall, propositions aim to present how things are instead of what things are.

Logic	Music
elements in a proposition	musical notes
sense (spatial arrangement of elements)	structure of musical notes
proposition	theme in music
propositional sign(projective relation to the world)	performance of the theme based on notes
fact(a propositional sign)	the performance
thought(a logical picture of facts)	the "framework" of performance: theme in music based on musical notes

With some fundamental knowledge of propositions, we introduce elementary propositions as the basic building blocks of propositions. Elementary propositions are the simplest propositions — one cannot find a smaller version of propositions that could constitute an elementary proposition.

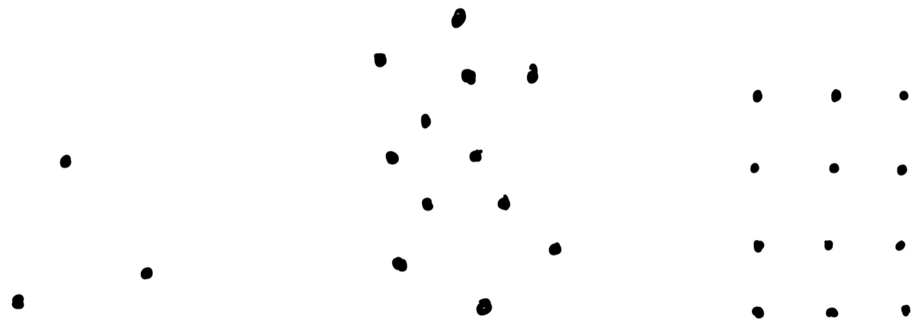
Elementary propositions are somewhat abstract ideas, because there cannot be any elementary proposition that contradicts an elementary proposition. It immediately follows from this characteristic that the negation of an elementary proposition is not an elementary proposition, since the unnegated version of the proposition would be a contradictory elementary proposition. In other words, ~p is not an elementary proposition because p, the unnegated version as an elementary proposition, contradicts it. When it involves two objects, it is unlikely that any proposition involving comparison could be an elementary proposition because it is easy to find a contradiction by switching. If we have A + [a verb that draws comparison] + B, we could then switch the position of A and B to have an elementary proposition that contradicts with the original one, as long as the comparison-verb does not allow A and B to be equal. In other words, if we have A < B then B < A will give us a contradiction. However, if we allow A <= B then B <= A will not raise a contradiction since we could have A = B to satisfy both. Elementary propositions are fundamental components of propositions and we usually could use p, q, r as references.

After building a proposition, to express its logical meaning, In Kant's system, a proposition relies on logical connectives, yet Wittgenstein expresses the idea that the truth-table is sufficient. All propositions are results of truth-operations on elementary propositions(5.3). One might notice that it means a proposition contains many possibilities of truth-conditions rather than asserting one. The truth-conditions, or truth-possibilities of a proposition, depend on the elementary propositions. Because each elementary proposition could be either true or false, definitely not both, 2 to the number of elementary propositions would be equal to the total possible number of truth-possibilities. For instance, if my proposition is composed of 3 elementary propositions, it will have 2^3, which gives us 8, number of truth-possibilities. Thus, no matter how long the statement is, for instance, if we have q or p, and, ~(p or q) and (p -> q), instead of manipulating the logical connectives, we could just observe their truth-tables. Although the latter one looks more complicated than the former, since both of them have only two elementary propositions, they will have the same number of truth-possibilities. Thus, observing the truth-tables adds a certain level of convenience.

Among all the truth-possibilities, Wittgenstein claims that tautologies and contradictions lack sense(4.461). The reason Wittgenstein provides is that both tautologies and contradictions contain no possibility, as the former represents all situations and the latter none. This reminds me of the word “trivial” in modern mathematics. For instance, given a set X, we would say it is trivial to state that the entire set X and the empty set are contained in a topology, because regardless of the construction of the topology, the entire set X and the empty set would always be in it. This might provide some clues to understand Wittgenstein’s reasoning, if we consider tautologies as a constant — they are true all the time — and contradictions as nothing as they are wrong and would not be able to correspond with reality. It is “trivial” to consider them because there is no surprise when we think about them, thus they lack sense.

However, Wittgenstein quickly adds that tautologies and contradictions but are not non-sensical because they are part of the symbolism as well as ‘0’ is part of the symbolism in arithmetic(4.4611). The same as lacking sense does not mean not non-sensical, being trivial does not mean being void or useless. In abstract algebra, we usually look for an identity element when we want to construct a group, ring or field, which are all important and convenient structures we could investigate later. A concept that corresponds with an identity element is an operation. For instance, addition as an operation has 0 as the identity element, because any number pluses 0 will still be the same number. Multiplication as an operation has 1 as the identity element, because any number multiply 1 will still be the same number. The identity element almost serves as a quiet foundation, and any element would not be influenced at all if they build on the foundation. At the same time, the foundation implies how we perceive the other elements. When the operation, the way we manipulate the elements, changes, the identity changes as well. Thus the reason that tautologies, all true, and contradictions, all false, are the identity, the quiet foundation, of other contingent propositions, might be that we care about all the different possibilities and not knowing the proposition is true or false provides the value of looking into it. If our attention is something else, then tautologies and contradictions would not be the foundation. Overall, my presumption is that tautologies and contradictions lack sense because we care more about the contingency in propositions to have further investigation, but they are not non-sensical because they serve as the foundation of our system.

Putting propositions together naturally reminds us of inference, on which Wittgenstein and Kant share similar views: a way of showing the hidden in premises. Kant thinks a conclusion only makes what was implicit in premises explicit. Similarly, Wittgenstein thinks the conclusion is already contained in the premises: a proposition affirms every proposition that follows from it(5.124). This reminds me of what John Von Neumann once said, "young man, in mathematics you don't understand things. You just get used to them." A proposition, in Wittgenstein's perspective, already contains what could follow from it, and thus we don't necessarily need the help from the laws of inference to deduce a proposition. It almost feels that Wittgenstein is suggesting that if we stare at a proposition long enough and really think about the logic and content of the proposition, we should be able to tell every implication of it. Most people can tell the number of dots on a screen immediately if the number is small, like 3. However, if there are, for example, 12 dots on a screen, it is hard to tel the number immediately, but we could if we are very used to the pattern of 12 dots being on a screen. Wittgenstein also proposes to write our propositions in different ways so that the relation between two propositions could be unmasked(5.1311). If we put 12 dots in 4 rows and 3 columns, then we could tell the number immediately as long as we know how to do basic multiplication. This shares a similar intention as Venn Diagram. We could recall that in Venn Diagram, after performing the operations which correspond with the premises, the conclusion is shown, effortlessly. Since the conclusion is shown after "trimming" the same diagram we started with, the conclusion was already in the diagram at the beginning. In general, both Kant and Wittgenstein seem to agree that what we could deduce was already hidden in the premises.



3, easy   12, hard, but doable   12, put in a way that’s easy

In conclusion, Wittgenstein aims to provide a more direct description of the essence behind all the names, propositions, logical connectives, and more. The logical connectives and laws of inference that mathematicians play around seem to be, to some extent, useless to Wittgenstein as one could just refer to the truth-table. Wittgenstein says, “what can be said at all can be said clearly”, and listing all the possibilities directly seems to be clearer than having connectives and drawing inference(3). The desire of speaking directly and clearly is so strong that some of his sentences would appear to be obvious in daily lives but confusing in a philosophical context, such as “what we cannot talk about we must pass over in silence” and “the only distinction between them, … is that they are different”(2.0233). By the end of the story, Wittgenstein draws his own picture of a logical system, and although I disagree with laws of inference having no sense, his system is weirdly convincing.