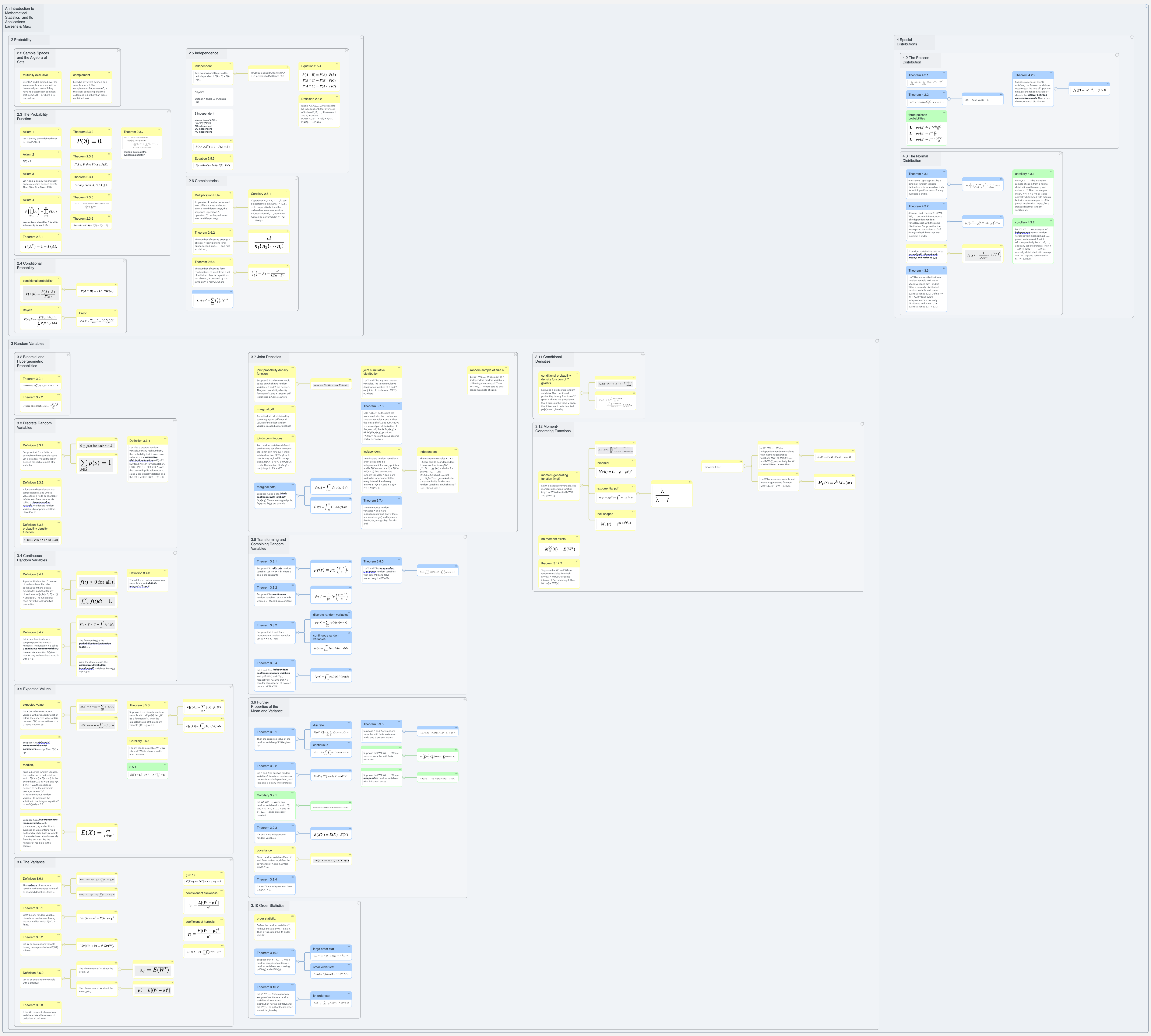
An Introduction to Mathematical Statistics and Its Applications - Larsens & Marx
2 Probability
2.2 Sample Spaces and the Algebra of Sets
mutually exclusive
Events A and B defined over the same sample space are said to be mutually exclusive if they have no outcomes in common–that is, if A ∩ B = ∅, where ∅ is the null set
complement
Let A be any event defined on a sample space S. The complement of A, written AC, is the event consisting of all the outcomes in S other than those contained in A.
2.3 The Probability Function
Axiom 1
Let A be any event defined over S. Then P(A) ≥ 0
Axiom 2
P(S) = 1
Axiom 3
Let A and B be any two mutually exclusive events defined over S. Then P(A ∪ B) = P(A) + P(B)
Axiom 4
$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$
intersections should be 0 for all Ai \intersect Aj for each i != j
Theorem 2.3.1
$P(A^C) = 1 - P(A).$
Theorem 2.3.2
$P(\emptyset) = 0.$
Theorem 2.3.3
If $A \subset B$, then $P(A) \le P(B)$.
Theorem 2.3.4
For any event A, $P(A) \le 1$.
Theorem 2.3.5
Theorem 2.3.6
$P(A \cup B) = P(A) + P(B) - P(A \cap B).$
Theorem 2.3.7
intuition: delete all the overlapping part till 1
2.4 Conditional Probability
conditional probability
$P(A|B) = \frac{P(A \cap B)}{P(B)}$
$P(A \cap B) = P(A|B)P(B)$
Baye's
$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$
Proof
$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{P(B)}$
2.5 Independence
independent
Two events A and B are said to be independent if P(A ∩ B) = P(A) · P(B).
P(A|B) can equal P(A) only if P(A ∩ B) factors into P(A) times P(B)
disjoint
union of A and B => P(A) plus P(B)
3 independent
intersection of ABC = P(A)*P(B)*P(C)
AB independent
BC independent
AC independent
$P(A^C \cup B^C) = 1 - P(A \cap B)$
Equation 2.5.3
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$
Equation 2.5.4
$P(A \cap B) = P(A) \cdot P(B)$
$P(B \cap C) = P(B) \cdot P(C)$
$P(A \cap C) = P(A) \cdot P(C)$
Definition 2.5.2
Events A1, A2, . . ., Anare said to be independent if for every set of indices i1, i2, . . ., ikbetween 1 and n, inclusive, P(Ai1∩ Ai2∩ · · · ∩ Aik) = P(Ai1) · P(Ai2) · · · · · P(Aik)
2.6 Combinatorics
Multiplication Rule
If operation A can be performed in m different ways and oper- ation B in n different ways, the sequence (operation A, operation B) can be performed in m · n different ways
Corollary 2.6.1
If operation Ai, i = 1, 2, . . . , k, can be performed in niways, i = 1, 2, . . . , k, respec- tively, then the ordered sequence (operation A1, operation A2, . . ., operation Ak) can be performed in n1· n2· · · · nkways
Theorem 2.6.2
The number of ways to arrange n objects, n1being of one kind, n2of a second kind, . . . , and nrof an rth kind,
$\frac{n!}{n_1!\,n_2!\cdots n_r!}$
Theorem 2.6.4
The number of ways to form combinations of size k from a set of n distinct objects, repetitions not allowed, is denoted by the symbols?n k ?ornCk, where
$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$
$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$
3 Random Variables
3.2 Binomial and Hypergeometric Probabilities
Theorem 3.2.1
Theorem 3.2.2
3.3 Discrete Random Variables
Definition 3.3.1
Definition 3.3.2
Definition 3.3.3 - probability density function
Definition 3.3.4
3.4 Continuous Random Variables
Definition 3.4.1
Definition 3.4.2
Definition 3.4.3
3.5 Expected Values
expected value
median
Theorem 3.5.3
Corollary 3.5.1
3.6 The Variance
Definition 3.6.1
Theorem 3.6.1
Theorem 3.6.2
Definition 3.6.2
Theorem 3.6.3
coefficient of skewness
coefficient of kurtosis
3.7 Joint Densities
joint probability density function
joint cumulative distribution
random sample of size n
marginal pdf
jointly continuous
marginal pdfs
Theorem 3.7.3
independent
Theorem 3.7.4
3.8 Transforming and Combining Random Variables
Theorem 3.8.1
Theorem 3.8.2
Theorem 3.8.3
Theorem 3.8.4
Theorem 3.8.5
discrete random variables
continuous random variables
3.9 Further Properties of the Mean and Variance
Theorem 3.9.1
discrete
continuous
Theorem 3.9.2
Corollary 3.9.1
Theorem 3.9.3
covariance
Theorem 3.9.4
Theorem 3.9.5
3.10 Order Statistics
order statistic
Theorem 3.10.1
large order stat
small order stat
Theorem 3.10.2
ith order stat
3.11 Conditional Densities
conditional probability density function of Y given x
Let X and Y be discrete random variables. The conditional probability density function of Y given x–that is, the probability that Y takes on the value y given that X is equal to x–is denoted pY|x(y) and given by
$p_{Y|x}(y) = P(Y = y \mid X = x) = \frac{p_{X,Y}(x,y)}{p_X(x)}$
3.12 Moment-Generating Functions
moment-generating function (mgf)
Let W be a random variable. The moment-generating function (mgf) for W is denoted MW(t) and given by
binomial
$M_X(t) = (1 - p + pe^t)^n$
exponential pdf
$M_Y(t) = E(e^{tY}) = \int_0^\infty e^{ty} \cdot \lambda e^{-\lambda y}\, dy$
$\frac{\lambda}{\lambda - t}$
bell shaped
$M_Y(t) = e^{\mu t + \sigma^2 t^2/2}$
rth moment exists
$M_W^{(r)}(0) = E(W^r)$
theorem 3.12.2
Suppose that W1and W2are random variables for which MW1(t) = MW2(t) for some interval of t's containing 0. Then fW1(w) = fW2(w).
Theorem 3.12.3
et W1,W2, . . . ,Wnbe independent random variables with moment-generating functions MW1(t), MW2(t), . . . , and MWn(t), respectively. Let W = W1+ W2+ · · · + Wn. Then
$M_W(t) = M_{W_1}(t) \cdot M_{W_2}(t) \cdots M_{W_n}(t)$
Let W be a random variable with moment-generating function MW(t). Let V = aW + b. Then
$M_V(t) = e^{bt} M_W(at)$
4 Special Distributions
4.2 The Poisson Distribution
Theorem 4.2.1
Theorem 4.2.2
$p_X(k) = P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$
E(X) = λand Var(X) = λ.
three poisson probabilities
1. $p_X(k) \doteq e^{-np}\frac{(np)^k}{k!}$
2. $p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!}$
3. $p_X(k) = e^{-\lambda T}\frac{(\lambda T)^k}{k!}$
Theorem 4.2.2
Suppose a series of events satisfying the Poisson model are occurring at the rate of λ per unit time. Let the random variable Y denote the interval between consecutive events. Then Y has the exponential distribution
$f_Y(y) = \lambda e^{-\lambda y}, \quad y > 0$
4.3 The Normal Distribution
Theorem 4.3.1
(DeMoivre-Laplace) Let X be a binomial random variable defined on n indepen- dent trials for which p = P(success). For any numbers a and b,
Theorem 4.3.2
(Central Limit Theorem) Let W1, W2, . . . be an infinite sequence of independent random variables, each with the same distribution. Suppose that the mean μ and the variance σ2of fW(w) are both finite. For any numbers a and b
A random variableY is said to be normally distributed with mean μ and variance σ2if
$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$
Theorem 4.3.3
Let Y1be a normally distributed random variable with mean μ1and variance σ2 1, and let Y2be a normally distributed random variable with mean μ2and variance σ2 2. Define Y = Y1+ Y2. If Y1and Y2are independent, Y is normally distributed with mean μ1+ μ2and variance σ2 1+ σ2 2.
corollary 4.3.1
LetY1,Y2, . . .,Ynbe a random sample of size n from a normal distribution with mean μ and variance σ2. Then the sample mean,¯Y =1 n n ? i=1 Yi, is also normally distributed with mean μ but with variance equal to σ2/n (which implies that ¯Y−μσ/√nis a standard normal random variable, Z).
corollary 4.3.2
Let Y1, Y2, . . ., Ynbe any set of independent normal random variables with means μ1, μ2, . . ., μnand variances σ2 1, σ2 2, . . ., σ2 n, respectively. Let a1, a2, . . ., anbe any set of constants. Then Y = a1Y1+ a2Y2+ · · · + anYnis normally distributed with mean μ = n ? i=1 aiμiand variance σ2= n ? i=1 a2 iσ2 i.